AV - 8214

MA / M. Sc. (maths) III - Sem 2015-16

Functional Analysis & its Applications - I

Q # 1:

(i) Theorem (1), pp 141, D. Somasundaram.

(ii) $x = (x_1, x_2, \ldots, x_n)$,

$$\|x\| = \left(\sum_{i=1}^{n} |x_i|^2\right)^{1/2}.$$

(iii) First prove the lemma for $1 < p < \infty$, $1 < q < \infty$ such that $\frac{1}{p} + \frac{1}{q} = 1$ and for $a, b \geq 0$,

$a^{1/p} \cdot b^{1/q} \leq \frac{a}{p} + \frac{b}{q}$. After this prove that

$$\sum_{i=1}^{n} |x_i y_i| \leq \|x\|_p \cdot \|y\|_q$$

where $\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{1/p}$.

(iv) Obtain

$$\|x+y\|_p \leq \|x\|_p + \|y\|_p$$

$1 < p < \infty$.

V) Set of all continuous real valued functions on a Banach space X. whose norm is defined by

$$\|f\| = \max\{|f(x)| : x \in X\}.$$

VI) Let X be a normed linear space and $\|\cdot\|_1$, $\|\cdot\|_2$ are two norms defined on X; then

$$m\|x\|_1 \le \|x\|_2 \le M\|x\|_1$$

where $m, M \ge 0$.

VII). Let X be a complete metric space. Let T be a mapping from X into itself satisfying $\forall x, y \in X$,

$$d(Tx, Ty) \le \alpha d(x, y)$$

where $\alpha \in [0, 1)$.

Let $x_0 \in X$ be an arbitrary pt. of X and the seq $\{x_n\}$ in X satisfying Picard iteration system, then T has a unique fixed point.

VIII) Assume T be a contraction map and show that if $x_n \to x$ in X, then $Tx_n \to Tx$.

IX) Assume the sequence in a nls X is a weakly cgt sequence with two limits x and y and finally show that $x=y$.

X) Define weak convergence in a nls. and by example s.t. it is weakly cgt. but not strongly cgt..

Q #2:- Theorem (1), pp-84 Somasundaram (ref.)

Q #3:- Th. 4.13.2, pp. 292, Erwin Kreyszig.

Q #4:- Ref. Theorem (2), pp-59, Somasundaram (ref.)

Q #5:- Ref. G.F. Simons, Somasundaram.

Q #6:- Theorem 5.12, pp. 300 Erwin Kreyszig.

Q #17:- Ref Theorem 1, pp. 156, Somasundaram & G.F. Simons.

Q #8:- Any One of $\frac{5.2.1}{pp.\ 309}$, $\frac{5.3.1}{pp\ 315}$, $\frac{5.4.1}{321}$

~~by~~ of Erwin Kreyszig

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